

## Practice Exam 1

1 An object is thrown from the top of a tall building. It's height, in feet, above the ground at time  $t$ , in seconds, is given by the equation  $f(t) = -16t^2 + 5t + 450$ .

a) At what time, to the nearest thousandth, does the object hit the ground?

b) Find the average velocity of the object over the following time intervals.

i) It's entire flight

ii) the last second of it's flight

iii) the last 0.1 second of its flight

iv) the last 0.01 second of its flight

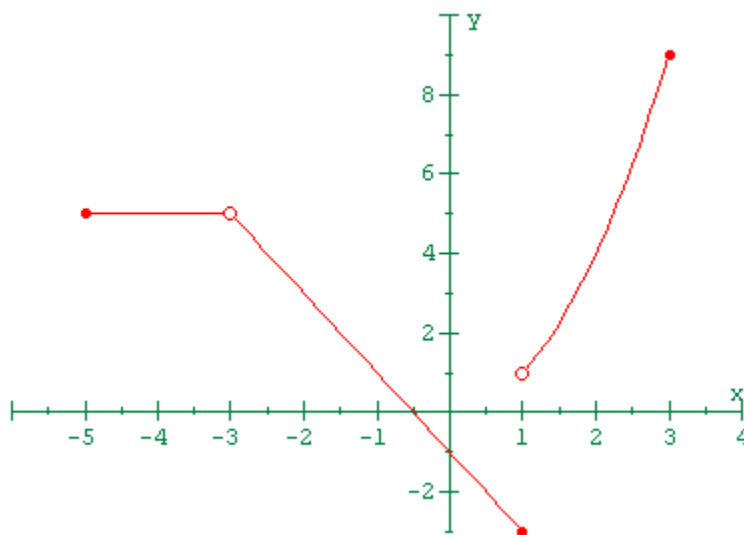
2) For the given graph of  $f(x)$  determine the following:

a)  $\lim_{x \rightarrow -3} f(x)$

b)  $\lim_{x \rightarrow 1^+} f(x)$

c)  $\lim_{x \rightarrow 1^-} f(x)$

d)  $\lim_{x \rightarrow 1} f(x)$



e) All points of discontinuity

f) At  $x = -3$ , is the function continuous from the left, from the right or neither?

g) At  $x = 1$ , is the function continuous from the left, from the right or neither?

3) Use your calculator to approximate the limit.

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{x^2 + 3x}$$

4) Find the limit if it exists.

a)  $\lim_{x \rightarrow 2} \sqrt{5x^2 - 3x + 2}$

b)  $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x^2 + x - 12}$

c)  $\lim_{x \rightarrow 2} \begin{cases} 3x - 5 & \text{if } x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases}$

d)  $\lim_{x \rightarrow 0^-} \frac{|x|}{x}$

5) If  $1 \leq f(x) \leq x^2 + 2x + 2$  for all  $x$ , find  $\lim_{x \rightarrow -1} f(x)$ . Justify your answer.

6) For the limit  $\lim_{x \rightarrow 3} 4x^2 - 10x + 2 = 8$ , find a value for  $\delta$  that corresponds to  $\varepsilon = 0.2$ .

8) Find all values of  $x$  where the function is discontinuous.

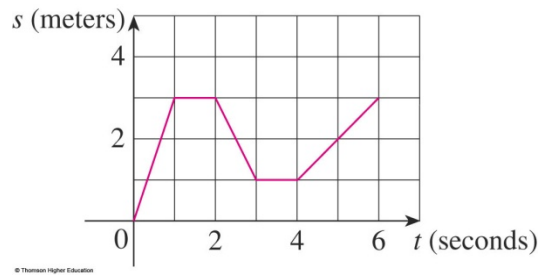
$$f(x) = \begin{cases} x+1 & \text{if } x \leq 1 \\ 1/x & \text{if } 1 < x < 3 \\ \sqrt{x-3} & \text{if } x \geq 3 \end{cases}$$

9) Find the value(s) of  $c$  that make the function continuous everywhere.

$$f(x) = \begin{cases} cx^2 & \text{if } x \leq 3 \\ cx+1 & \text{if } x > 3 \end{cases}$$

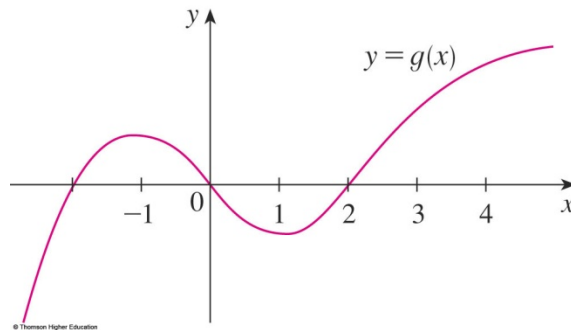
10) For the function  $f(x) = \frac{2x^2 + x - 1}{x^2 - x - 2}$ , find the vertical and horizontal asymptotes.

11. A particle moves along a straight line. It's position is given by the graph below. Draw a graph of the velocity function.



12. For the function  $g$  whose graph is given, arrange the following numbers in increasing order and explain your reasoning.

$$0, \quad g'(-2), \quad g'(0), \quad g'(2), \quad g'(4)$$

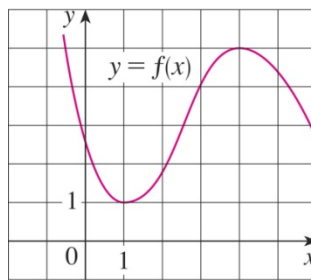


13. Using the definition of the derivative, if  $f(x) = x^2 - 3x$ , find  $f'(2)$ .

14. Using the definition of the derivative, if  $f(x) = \sqrt{2x-3}$ , find  $f'(x)$ .

15. Use the graph to estimate the value of each derivative, then sketch the graph of  $f'$ .

- a)  $f''(0)$
- b)  $f''(1)$
- c)  $f''(2)$
- d)  $f''(3)$
- e)  $f''(4)$
- f)  $f''(5)$



16. Let  $P(t)$  be the percentage of Americans under the age of 18 at time  $t$ . The table gives values of this function in census years from 1950 to 2000.

- a) What is the meaning of  $P'(t)$ ?
- b) Construct a table of estimated values for  $P'(t)$ .
- c) Graph  $P$  and  $P'$ .
- d) How would it be possible to get more accurate values for  $P'(t)$ ?

$t$	$P(t)$	$P'(t)$
1950	31.1	XXX
1960	35.7	
1970	34.0	
1980	28.0	
1990	25.7	
2000	25.7	XXX

Answers:

1a)  $t = 5.462$

1b) i) -82.39ft/sec, ii) -153.76ft/sec, iii) -167.93ft/sec, iv) -167.11ft/sec

2a) 5, b) 1, c) -3, d) DNE,

e) discontinuous at  $x = -3, 1$

f) the function is discontinuous from both sides at  $x = -3$

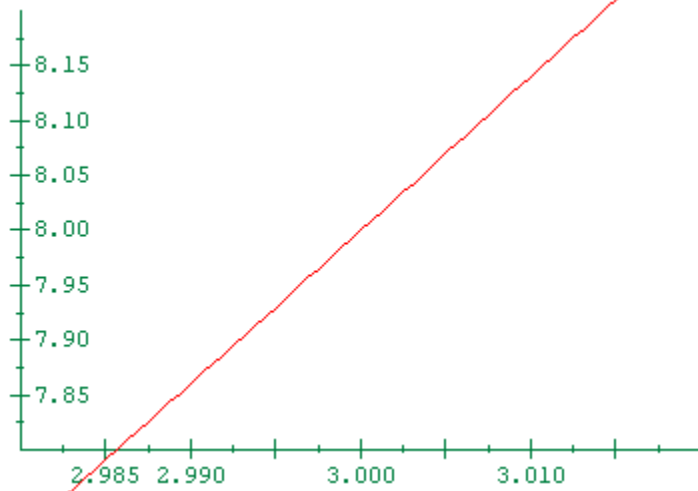
g) the function is continuous from the left at  $x = 1$  but not from the right

3) 2.3333

4) a) 4, b)  $-\frac{1}{7}$ , c) DNE, d) -1

5) Since  $\lim_{x \rightarrow -1} 1 = 1$  and  $\lim_{x \rightarrow -1} x^2 + 2x + 2 = 1$  and  $1 \leq f(x) \leq x^2 + 2x + 2$ , then by the Squeeze Theorem,  
$$\lim_{x \rightarrow -1} f(x) = 1$$

6) Using a window of 2.98 to 3.02 for  $x$  and 7.8 to 8.2 for  $y$ , I got the graph below. The values for  $x$  I got were 2.986 and 3.014. So we get  $\delta = 0.014$ .



Let  $\delta = \varepsilon$ , then  $|f(x) - 5| < \varepsilon$  when  $|x - 2| < \delta$ .

8) Discontinuous at  $x = 1, 3$  since the piecewise curves don't meet each other, but nowhere else since each function is continuous where it's defined.

9) We need the pieces to match when we plug in  $x = 3$ . So

$$c(3)^2 = c(3) + 1$$

$$9c = 3c + 1$$

$$6c = 1$$

$$c = \frac{1}{6}$$

10)  $\frac{2x^2 + x - 1}{x^2 + x - 2} = \frac{(x+1)(2x-1)}{(x+2)(x+1)} = \frac{2x-1}{x+2}$ . Since  $x+1$  cancels,  $x = -1$  is not a vertical asymptote. The

only vertical asymptote is at  $x = -2$ . The horizontal asymptotes are at

$$\lim_{x \rightarrow \infty} \frac{2x^2 + x - 1}{x^2 + x - 2} = \lim_{x \rightarrow \infty} \frac{2 + 1/x - 1/x^2}{1 + 1/x - 2/x^2} = \frac{2}{1} = 2$$

$$\lim_{x \rightarrow -\infty} \frac{2x^2 + x - 1}{x^2 + x - 2} = \lim_{x \rightarrow -\infty} \frac{2 + 1/x - 1/x^2}{1 + 1/x - 2/x^2} = \frac{2}{1} = 2$$

11.

12.  $g'(0), 0, g'(4), g'(2), g'(-2)$

13.  $f'(2) = 1$

14.  $f'(x) = \frac{1}{\sqrt{2x-3}}$

15.

a)  $f''(0) = -3$

b)  $f''(1) = 0$

c)  $f''(2) = 1$

d)  $f''(3) = 1.5$

e)  $f''(4) = 0$

f)  $f''(5) = -1$



16. a)  $P'(t)$  represents the instantaneous rate of change of the population in the year  $t$ .

b)

$t$	$P'(t)$
1960	0.145
1970	-0.385
1980	-0.415
1990	-0.115

c)

d) Take measurements more frequently.