Cuyamaca College
Math 180

Name:________________________
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Practice Exam 2

1. Find the derivative and second derivative of the following functions.

a) $y = \sqrt[3]{x}$

b) $f(x) = 6t^3 - 5t^2 + 7t - 3$

c) $y = (3x^2 - 5)^{17}$

d) $f(x) = x^3 e^x$

2. Find the derivative of the following functions

a) $y = \frac{x+1}{x^3+x-2}$

b) $f(x) = \frac{x^2-2\sqrt{x}}{x}$

c) $y = e^x \sin x$

d) $f(x) = \ln(x^2 + 10)$

e) $y = \frac{1-\sec x}{\tan x}$

f) $f(x) = \frac{\ln x}{x-\ln x}$

g) $y = x^{\cos x}$

h) $f(x) = \frac{3x^5(x^5-4)(x+7)^{12}}{\sqrt{x^9-4}}$

i) $y = e^{\cos(x^2+1)}$

j) $f(x) = \tan^{-1}(4x^3 - 5x^2)$

k) $f(x) = a^{(x^2-5x)}$

m) $f(x) = \log_a(\cos x)$

n) $f(x) = \csc(\tan x)$

o) $y = \sin^{-1}(x^2 + 1)$

p) $y = e^{\sin(3x)}$

q) $y = \tan^{-1} x \sin^{-1} x$

r) $y = \cosh(x^2)\sinh(x^2)$

3) Find the equation of the tangent line to the curve at the point $(3, 9e^3)$.

$$y = x^2 e^x$$

4) Find *dy/dx* by implicit differentiation.

a) $y^5 + x^2y^3 = 1 + ye^x$

b) $x^2 + y^2 = 1$

5) Use implicit differentiation to find the equation of the tangent line on the curve at the point (1, 2).

$$x^2 + 2xy - y^2 + x = 2$$

6) Using the fact that $\frac{d}{dx}(\sin x) = \cos x$ and that $\frac{d}{dx}(\cos x) = -\sin x$, derive the formula for

$$\frac{d}{dx}(\cot x)$$

7) Use implicit differentiation to derive the formula for

$$\frac{d}{dx}(\sec^{-1} x)$$

Answers:

1) a) $y' = \frac{1}{3}x^{-2/3}$

$y'' = -\frac{2}{9}x^{-5/3}$

b) $f'(x) = 18t^2 - 10t + 7$

$f''(x) = 36t - 10$

c) $y' = 102x(3x^2-5)^{16}$

$y'' = 9792x^2(3x^2-5)^{15} + 102(3x^2-5)^{16}$

d) $f'(x) = (x^3+3x^2)e^x$

$f''(x) = (x^3+6x^2+6x)e^x$

2) a) $y' = \frac{-2x^3-3x^2-3}{(x^3+x-2)^2}$

b) $f'(x) = 1 + x^{-3/2}$

c) $y' = e^x \cos x + e^x \sin x$

d) $f'(x) = \frac{2x}{x^2+10}$

e) $y' = \frac{-\sec x \tan^2 x - \sec^2 x + \sec^3 x}{\tan^2 x}$

f) $f'(x) = \frac{1-\ln x}{(x-\ln x)^2}$

g) $y' = x^{\cos x}\left(\frac{\cos x}{x} - \ln x \sin x\right)$

h) $f'(x) = \left(\frac{3x^5(x^5-4)(x+7)^{12}}{\sqrt{x^9-4}}\right)\left(\frac{5}{x} + \frac{5x^4}{x^5-4} + \frac{12}{x+9} - \frac{9x^8}{2(x^9-4)}\right)$

i) $y' = -2x\sin(x^2+1)e^{\cos(x^2+1)}$

j) $f'(x) = \frac{12x^2-10x}{(4x^3-5x^2)^2+1}$

k) $f'(x) = a^{(x^2-5x)} \ln a\,(2x-5)$

m) $f'(x) = \frac{1}{\cos x \ln a}(-\sin x)$

n) $f'(x) = -\csc(\tan x)\cot(\tan x)\sec^2 x$

o) $y' = \frac{1}{\sqrt{1-(x^2+1)^2}}(2x)$

p) $y' = e^{\sin(3x)}\cos(3x)(3)$

q) $y' = \frac{1}{1+x^2}\sin^{-1} x + \tan^{-1} x \frac{1}{\sqrt{1-x^2}}$

r) $y' = 2x\cosh^2(x^2) + 2x\sinh^2(x^2)$

3) $m = 15e^3, \qquad y = 15e^3 x - 36e^3$

4) a) $\frac{dy}{dx} = \frac{ye^x - 2xy^3}{5y^4 + 3x^2y^2 - e^x}$

b) $\frac{dy}{dx} = -\frac{x}{y}$

5) $\dfrac{dy}{dx}=\dfrac{-2x-2y-1}{2x-2y} \qquad m=\dfrac{7}{2} \qquad y=\dfrac{7}{2}x-\dfrac{3}{2}$

6)
$$y=\cot x=\frac{\cos x}{\sin x}$$
$$y'=\frac{\sin x(-\sin x)-\cos x\cos x}{\sin^2 x}=\frac{-(\sin^2 x+\cos^2 x)}{\sin^2 x}=-\frac{1}{\sin^2 x}=-\csc^2 x$$

7)
$$y=\sec^{-1} x$$
$$x=\sec y$$
$$1=\sec y\tan y\frac{dy}{dx}$$
$$\frac{dy}{dx}=\frac{1}{\sec y\tan y}=\frac{1}{\sec y\sqrt{\sec^2 y-1}}=\frac{1}{x\sqrt{x^2-1}}$$