

Practice Exam 3

1. Car A is travelling west toward an intersection at 40 mph. Car B is travelling north toward the same intersection at 60 mph. At what rate are the cars approaching each other when car A is 3 miles from the intersection and car B is 4 miles from the intersection?

2. A spherical balloon is being filled with water at the rate of 100 cm^3 per second. At what rate is the radius increasing when the radius is 15 cm?

4. Find the critical values of the following functions.

a) $f(x) = 3x^4 + 4x^3 - 6x^2$

b) $y = \frac{p-1}{p^2+4}$

5. Find the absolute maximum and minimum values of f on the given interval.

a) $f(x) = (x^2 - 1)^3$, $[-1, 2]$

b) $f(t) = t - \ln t$, $[\frac{1}{2}, 2]$

6. Verify that the function satisfies the conditions of the Mean Value Theorem, and then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$f(x) = x^3 + x - 1$, $[0, 2]$

7. Let $f(x) = \frac{x^2}{x^2 - 9}$.

- a) Find intervals of increasing and decreasing.
- b) Find all local maximums and minimums.
- c) Find all intervals of concavity and inflection points.
- d) Sketch the curve.

8. Find the point on the line $y = 4x + 7$ that is closest to the origin.

9. The rate (in mg carbon/m³/h) at which photosynthesis takes place for a species of phytoplankton is modeled by the function

$$P = \frac{100x}{x^2 + x + 4}$$

where x is the light intensity (measured in thousands of foot candles). For what light intensity is P a maximum?

Answers:

1) 72 mph

2) 0.035 cm/sec

4) a) $x = 0, \frac{-1 \pm \sqrt{5}}{2}$, b) $1 \pm \sqrt{5}$

5) a) max: (2, 1), min: (0, -1)

b) max: (2, 1.307), min: (1, 1)

6) $c = \sqrt{\frac{4}{3}}$

7) a) Critical values: -3, 0, 3

Increasing: $(-\infty, -3)$ and $(-3, 0)$

Decreasing: $(0, 3)$ and $(3, \infty)$

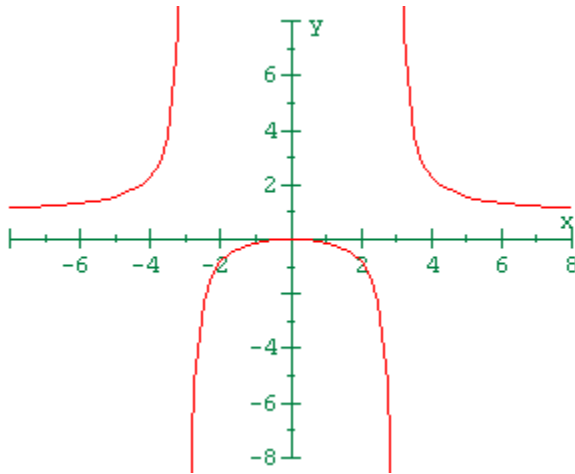
b) Local max: (0,0)

c) Inflection points: -3, 3

Concave up: $(-\infty, -3)$ and $(3, \infty)$

Concave down: $(-3, 3)$

d)



8) $\left(\frac{-28}{17}, \frac{7}{17}\right)$

9) 2 thousand foot candles