Math 180
Cuyamaca College

Name:______________________________
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Practice Final

1 An object is thrown from the top of a tall building. It's height, in feet, above the ground at time *t*, in seconds, is given by the equation $f(t) = -16t^2 + 5t + 450$.

a) At what time, to the nearest thousandth, does the object hit the ground?

b) What is the velocity of the object after 3 seconds?

c) What is the velocity of the object when it hits the ground?

d) What is the maximum height of the object

2) Find the limit if it exists.

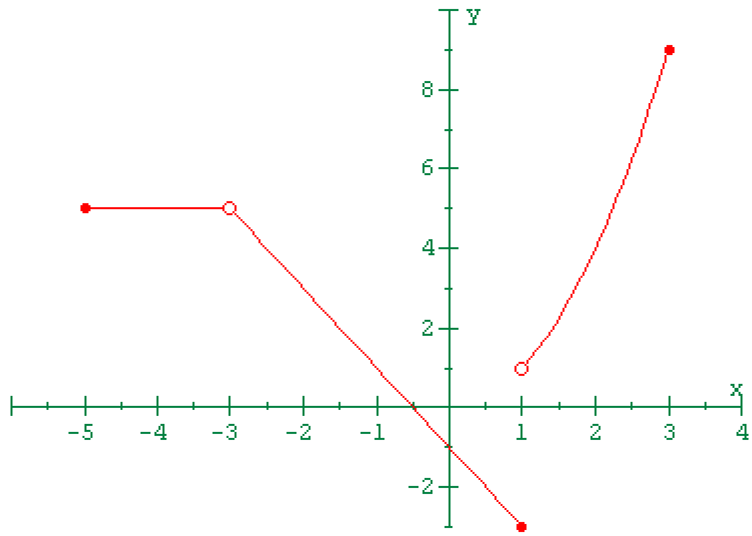
a) $\lim_{x \to 2} \sqrt{5x^2 - 3x + 2}$

b) $\lim_{x \to 3} \frac{x^2 - 7x + 12}{x^2 + x - 12}$

c) $\lim_{x \to 2} \frac{|2 - x|}{2 - x}$

d) $\lim_{x \to 0^-} \frac{|x|}{x}$

3) For the given graph of $f(x)$ determine the following:



a) $\lim_{x \to -3} f(x)$

b) $\lim_{x \to 1^+} f(x)$

c) $\lim_{x \to 1^-} f(x)$

d) $\lim_{x \to 1} f(x)$

e) All points of discontinuity

f) At $x = -3$, is the function continuous from the left, from the right or neither?

g) At $x = 1$, is the function continuous from the left, from the right or neither?

4) For the limit $\lim_{x \to 3} 4x^2 - 10x + 2 = 8$, find a value for δ that corresponds to $\varepsilon = 0.2$. List the values you use for your window and sketch what the graph looks like in that window.

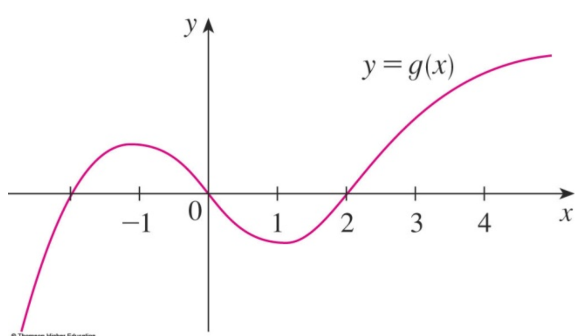
5) Find all values of x where the function is discontinuous.

$$f(x)=\begin{cases} x+1 & \text{if } x\le 1 \\ 1/x & \text{if } 1<x<3 \\ \sqrt{x-3} & \text{if } x\ge 3 \end{cases}$$

6) For the function $f(x)=\dfrac{2x^2+x-1}{x^2+x-2}$, find the vertical and horizontal asymptotes.

7. For the function g whose graph is given, arrange the following numbers in increasing order and explain your reasoning.

$$0, g'(-2), g'(0), g'(2), g'(4)$$



8. Using the definition of the derivative, if $f(x)=x^2-3x$, find $f'(2)$.

9. Find the derivative of the following functions, and the second derivative for problems a, b, c and f.

a) $y = \sqrt[3]{x}$

b) $f(x) = 6t^3 - 5t^2 + 7t - 3$

c) $y = (3x^2 - 5)^{17}$

d) $f(x) = \frac{x^2 - 2\sqrt{x}}{x}$

e) $y = \frac{x+1}{x^3+x-2}$

f) $f(x) = x^3 e^x$

g) $y = e^x \sin x$

h) $f(x) = \ln(x^2 + 10)$

i) $y = \frac{1-\sec x}{\tan x}$

j) $f(x) = \frac{\ln x}{x - \ln x}$

k) $y = x^{\cos x}$

l) $f(x) = \frac{3x^5(x^5-4)(x+7)^{12}}{\sqrt{x^9-4}}$

m) $y = e^{\cos(x^2+1)}$

n) $f(x) = \tan^{-1}(4x^3 - 5x^2)$

o) $y = \cosh(x^2 - 2x + 1)$

p) $y = \operatorname{sech} x \tanh x$

10. Let P(t) be the percentage of Americans under the age of 18 at time *t*. The table gives values of this function in census years from 1950 to 2000.

a) What is the meaning of P'(t)?
b) Construct a table of estimated values for P'(t).
c) Graph P and P'.
d) How would it be possible to get more accurate values for P'(t)?

t	P(t)
1950	31.1
1960	35.7
1970	34.0
1980	28.0
1990	25.7
2000	25.7

11. Find the equation of the tangent line to the curve at the point (3, 0).

$$y = x^2 \ln(x-2)$$

12. Find *dy/dx* by implicit differentiation.

a) $y^5 + x^2y^3 = 1 + ye^x$

b) $x^2 + y^2 = 1$

13. Use implicit differentiation to find the equation of the tangent line on the curve at the point (1, 2).

$$x^2 + 2xy - y^2 + x = 2$$

14. Car A is travelling west toward an intersection at 40 mph. Car B is travelling north toward the same intersection at 60 mph. At what rate are the cars approaching each other when car A is 3 miles from the intersection and car B is 4 miles from the intersection?

15. A spherical balloon is being filled with water at the rate of 100 cm^3 per second. At what rate is the radius increasing when the radius is 15 cm?

16. Find the critical values of the following functions.

a) $f(x) = 3x^4 + 4x^3 - 6x^2$

b) $y = \dfrac{p-1}{p^2+4}$

17. Find the absolute maximum and minimum values of f on the given interval.

a) $f(x) = (x^2 - 1)^3, \quad [-1, 2]$

b) $f(t) = t - \ln t, \quad [\frac{1}{2}, 2]$

18. Let $f(x) = 4x^3 + 3x^2 - 6x + 1$.

a) Find the intervals on which f is increasing or decreasing.

b) Find the local maximum and minim values of *f*.

c) Find the intervals of concavity and the inflection points.

d) Use the information from a-c to make a rough sketch of the graph.

19. Find the point on the line $y = 4x + 7$ that is closest to the origin.

20. The rate (in mg carbon/m^3/h) at which photosynthesis takes place for a species of phytoplankton is modeled by the function

$$P = \frac{100x}{x^2 + x + 4}$$

where *x* is the light intensity (measured in thousands of foot candles). For what light intensity is P a maximum?

21. Evaluate the following integrals.

a) $\int_3^6 4x^3 - 5x^2 + 2\,dx$

b) $\int_1^4 \frac{x-1}{\sqrt{x}}\,dx$

c) $\int_1^3 \frac{1}{2x}\,dx$

d) $\int (x-2)(x+1)\,dx$

e) $\int \csc^2 t - 2e^t\,dt$

f) $\int e^{\cos t} \sin t\,dt$

g) $\int x\cos(x^2)\,dx$

22) Approximate the following integral using the left-hand rule with $n = 4$.

$$\int_0^1 xe^x dx$$

23) Calculate the area between the curves $y = x^2$ and $y = e^x$, between $x = 1$ and $x = 3$.

24) Find the area enclosed by the curves $y = x$ and $y = x^2$.

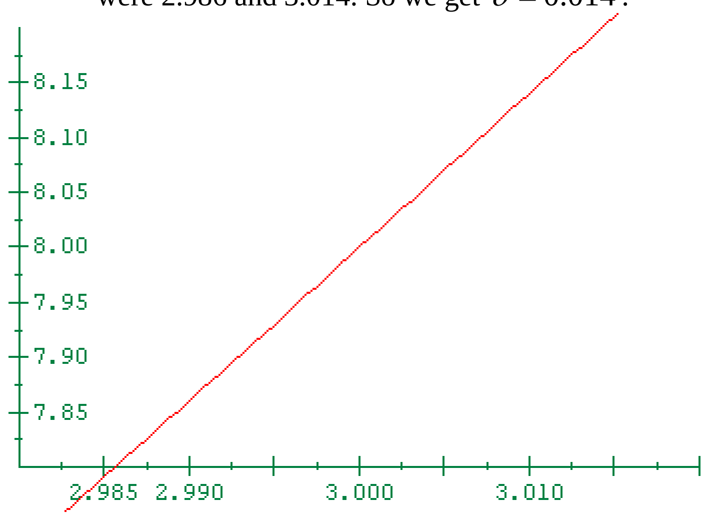
Answers:

1a) $t = 5.462$

2) a) 4, b) $-\frac{1}{7}$, c) DNE, d) -1

3 a) 5, b) 1, c) -3, d) DNE,
e) discontinuous at $x = -3, 1$
f) the function is discontinuous from both sides at $x = -3$
g) the function is continuous from the left at $x = 1$ but not from the right

4) Using a window of 2.98 to 3.02 for x and 7.8 to 8.2 for y, I got the graph below. The values for x I got were 2.986 and 3.014. So we get $\delta = 0.014$.



5) Discontinuous at $x = 1, 3$ since the piecewise curves don't meet each other, but nowhere else since each function is continuous where it's defined.

6) $\frac{2x^2 + x - 1}{x^2 + x - 2} = \frac{(x+1)(2x-1)}{(x+2)(x-1)}$. Since nothing cancels, the vertical asymptotes are when the denominator is 0, which is at $x = -2, 1$. The horizontal asymptotes are at

$$\lim_{x \to \infty} \frac{2x^2 + x - 1}{x^2 + x - 2} = \lim_{x \to \infty} \frac{2 + 1/x - 1/x^2}{1 + 1/x - 2/x^2} = \frac{2}{1} = 2$$

$$\lim_{x \to -\infty} \frac{2x^2 + x - 1}{x^2 + x - 2} = \lim_{x \to -\infty} \frac{2 + 1/x - 1/x^2}{1 + 1/x - 2/x^2} = \frac{2}{1} = 2$$